Sandwich classification for classical-like groups over commutative rings

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Theorem (Wilson-Golubchik, 1972/73)

Let H be a subgroup of $GL_n(R)$. Then H is normalized by $E_n(R)$ iff

$$E_n(R, I) \subseteq H \subseteq C_n(R, I)$$

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for some ideal I of R.

The ideal I in the theorem above is uniquely determined, namely

$$I = I(H) := \{x \in R \mid t_{kl}(x) \in H \text{ for some } k \neq l\}.$$

Let $\sigma \in GL_n(R)$ and set $H := {}^{E_n(R)}\langle \sigma \rangle$, i.e. H is the smallest subgroup of $GL_n(R)$ which contains σ and is normalized by $E_n(R)$. By the Sandwich Classification Theorem one has $H \subseteq C_n(R, I)$ where $I = I(H)$. That amounts to saying that for any $i \neq j$ and $k \neq l$ the matrices $t_{kl}(\sigma_{ij})$ and $t_{kl}(\sigma_{ii} - \sigma_{jj})$ can be expressed as products of matrices of the form ${}^{\epsilon}\sigma^{\pm 1}$ where $\epsilon \in E_n(R)$.

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Problem: Find such expressions of the matrices $t_{kl}(\sigma_{ij})$ and $t_{kl}(\sigma_{ii} - \sigma_{jj})$ with a small number of factors.

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Lemma

The relations

$$t_{ij}(x)t_{ij}(y) = t_{ij}(x+y), \tag{R1}$$

$$[t_{ij}(x), t_{hk}(y)] = e \text{ and} \tag{R2}$$

$$[t_{ij}(x), t_{jk}(y)] = t_{ik}(xy) \tag{R3}$$

hold where $i \neq k, j \neq h$ *in* (R2) *and* $i \neq k$ *in* (R3).

Definition

Let I be an ideal of R. Then the canonical map $R \to R/I$ induces a group homomorphism $\phi : GL_n(R) \to GL_n(R/I)$. The preimage of $Center(GL_n(R/I))$ under ϕ is called *full congruence subgroup of level I* and is denoted by $C_n(R, I)$.

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Remark

Let $\sigma \in GL_n(R)$ and I be an ideal of R. Then $\sigma \in C_n(R, I)$ iff $\sigma_{ij}, \sigma_{ii} - \sigma_{jj} \in I$ for any $i \neq j$.

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Theorem

Let I be an ideal of R. Then the Standard Commutator Formulas

$$[C_n(R, I), E_n(R)] = [E_n(R, I), E_n(R)] = E_n(R, I)$$

hold.

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Let $\sigma \in GL_n(R)$. Then a matrix of the form ${}^{\epsilon}\sigma^{\pm 1}$ where $\epsilon \in E_n(R)$ is called an *elementary σ-conjugate*.

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Theorem (RP, 2017)

Let $\sigma \in GL_n(R)$, $i \neq j$ and $k \neq l$. Then

(i) $t_{kl}(\sigma_{ij})$ *is a product of* 8 *elementary σ-conjugates and*

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Proof.

(i) One checks easily that

$$[t_{12}(1), {}^{\tau^{-1}}[t_{32}(1), [\tau, \sigma]]] = t_{32}(\sigma_{23})$$

where $\tau = t_{21}(-\sigma_{23})t_{31}(\sigma_{22})$.

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$$[t_{12}(1), {}^{\tau^{-1}}[t_{32}(1), [\tau, \sigma]]] = t_{32}(\sigma_{23})$$

where $\tau = t_{21}(-\sigma_{23})t_{31}(\sigma_{22})$. This implies that $t_{32}(\sigma_{23})$ is a product of 8 elementary σ-conjugates. Using the fact that $E_n(R)$ contains certain monomial matrices one can deduce (i).

(ii) Clearly the entry of ${}^{t_{ji}(1)}\sigma$ at position (j, i) equals $\sigma_{ii} - \sigma_{jj} + \sigma_{ji} - \sigma_{ij}$.

(ii) Clearly the entry of ${}^{t_{ji}(1)}\sigma$ at position (j,i) equals $\sigma_{ii}-\sigma_{jj}+\sigma_{ji}-\sigma_{ij}$. Applying (i) to ${}^{t_{ji}(1)}\sigma$ one gets that $t_{kl}(\sigma_{ii}-\sigma_{jj}+\sigma_{ji}-\sigma_{ij})$ is a product of 8 elementary σ-conjugates (clearly any elementary ${}^{t_{ji}(1)}\sigma$-conjugate is also an elementary σ-conjugate).

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(ii) Clearly the entry of ${}^{t_{ji}(1)}\sigma$ at position (j, i) equals $\sigma_{ii} - \sigma_{jj} + \sigma_{ji} - \sigma_{ij}$. Applying (i) to ${}^{t_{ji}(1)}\sigma$ one gets that $t_{kl}(\sigma_{ii} - \sigma_{jj} + \sigma_{ji} - \sigma_{ij})$ is a product of 8 elementary σ-conjugates (clearly any elementary ${}^{t_{ji}(1)}\sigma$-conjugate is also an elementary σ-conjugate). Applying (i) to σ one gets that $t_{kl}(\sigma_{ij} - \sigma_{ji}) = t_{kl}(\sigma_{ij})t_{kl}(-\sigma_{ji})$ is a product of 16 elementary σ-conjugates. It follows that $t_{kl}(\sigma_{ii} - \sigma_{jj}) = t_{kl}(\sigma_{ii} - \sigma_{jj} + \sigma_{ji} - \sigma_{ij})t_{kl}(\sigma_{ij} - \sigma_{ji})$ is a product of 24 elementary σ-conjugates. □

Corollary

Let H be a subgroup of $GL_n(R)$. Then H is normalized by $E_n(R)$ iff

$$E_n(R, I) \subseteq H \subseteq C_n(R, I) \tag{1}$$

for some ideal I of R.

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Proof.

($\Rightarrow$) Suppose that H is normalized by $E_n(R)$. Set $I = I(H)$. Then clearly $E_n(R,I) \subseteq H$. Let $\sigma \in H$, $i \neq j$ and $k \neq l$. By the previous theorem, $T_{kl}(\sigma_{ij})$ and $T_{kl}(\sigma_{ii} - \sigma_{jj})$ are products of matrices of the form ${}^{\epsilon}\sigma^{\pm 1}$ where $\epsilon \in E_n(R)$. Hence $T_{kl}(\sigma_{ij}), T_{kl}(\sigma_{ii} - \sigma_{jj}) \in H$ which implies $\sigma_{ij}, \sigma_{ii} - \sigma_{jj} \in I$. Thus $H \subseteq C_n(R,I)$.

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($\Rightarrow$) Suppose that H is normalized by $E_n(R)$. Set $I = I(H)$. Then clearly $E_n(R,I) \subseteq H$. Let $\sigma \in H$, $i \neq j$ and $k \neq l$. By the previous theorem, $T_{kl}(\sigma_{ij})$ and $T_{kl}(\sigma_{ii} - \sigma_{jj})$ are products of matrices of the form ${}^{\epsilon}\sigma^{\pm 1}$ where $\epsilon \in E_n(R)$. Hence $T_{kl}(\sigma_{ij}), T_{kl}(\sigma_{ii} - \sigma_{jj}) \in H$ which implies $\sigma_{ij}, \sigma_{ii} - \sigma_{jj} \in I$. Thus $H \subseteq C_n(R,I)$.
($\Leftarrow$) Suppose that (1) holds for some ideal I. Then

$$[H, E_n(R)] \subseteq [C_n(R,I), E_n(R)] = E_n(R,I) \subseteq H.$$

Hence H is normalized by $E_n(R)$. □

Definition

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\begin{aligned}
q : V &\to R \\
v &\mapsto v^t \begin{pmatrix} 0 & p \\ 0 & 0 \end{pmatrix} v = \sum_{i=1}^{n} v_i v_{-i}
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where $p \in M_n(R)$ is the matrix with ones on the skew diagonal and zeros elsewhere.

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where $p \in M_n(R)$ is the matrix with ones on the skew diagonal and zeros elsewhere. The subgroup

$$O_{2n}(R) := \{\sigma \in GL_{2n}(R) \mid q(\sigma v) = q(v)\ \forall v \in V\}$$

of $GL_{2n}(R)$ is called *(even-dimensional) orthogonal group*.

Lemma

Let $\sigma \in GL_{2n}(R)$*. Then* $\sigma \in O_{2n}(R)$ *iff*

(i) $\sigma^{-1}_{ij} = \sigma_{-j,-i} \; \forall i,j \in \{1,...,n,-n,...,-1\}$ *and*

(ii) $q(\sigma_{*j}) = 0 \; \forall j \in \{1,...,n,-n,...,-1\}$.

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Lemma

Let $\sigma \in O_{2n}(R)$ *and* $k \in \{1, ..., n, -n, ..., -1\}$. *Then the statements below are true.*

(i) *If* σ_{*k} *is trivial then* $\sigma_{-k,*}$ *is trivial.*

(ii) *If* σ_{k*} *is trivial then* $\sigma_{*,-k}$ *is trivial.*

Definition

If $i, j \in \{1, ..., n, -n, ..., -1\}$ such that $i \neq \pm j$ and $x \in R$, then the matrix

$$T_{ij}(x) := e + xe^{ij} - xe^{-j,-i} \in O_{2n}(R)$$

is called an *elementary orthogonal transvection*. The subgroup of $O_{2n}(R)$ generated by all elementary orthogonal transvections is called *elementary orthogonal group* and is denoted by $EO_{2n}(R)$.

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Lemma

The relations

$$T_{ij}(x) = T_{-j,-i}(-x), \tag{R1}$$

$$T_{ij}(x)T_{ij}(y) = T_{ij}(x+y), \tag{R2}$$

$$[T_{ij}(x), T_{hk}(y)] = e, \tag{R3}$$

$$[T_{ij}(x), T_{jk}(y)] = T_{ik}(xy), \tag{R4}$$

$$[T_{ij}(x), T_{j,-i}(y)] = e \tag{R5}$$

hold where $h \neq j, -i$ and $k \neq i, -j$ in (R3) *and $i \neq \pm k$ in* (R4).

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Let $\sigma \in O_{2n}(R)$ *and* I *be an ideal of* R*. Then* $\sigma \in CO_{2n}(R, I)$ *iff* $\sigma_{ij}, \sigma_{ii} - \sigma_{jj} \in I$ *for any* $i \neq j$*.*

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Theorem

Let I *be an ideal of* R. *Then the Standard Commutator Formulas*

$$[CO_{2n}(R, I), EO_{2n}(R)] = [EO_{2n}(R, I), EO_{2n}(R)] = EO_{2n}(R, I)$$

hold.

Definition

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(ii) $T_{kl}(\sigma_{i,-i})$ *is a product of* 16 *elementary orthogonal σ-conjugates,*

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where $\tau = T_{21}(-\sigma_{23})T_{31}(\sigma_{22})T_{2,-3}(\sigma_{2,-1})$. □

Corollary

Let H be a subgroup of $O_{2n}(R)$. Then H is normalized by $EO_{2n}(R)$ iff

$$EO_{2n}(R, I) \subseteq H \subseteq CO_{2n}(R, I)$$

for some ideal I of R.

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Theorem (RP, 2017)

Let $\sigma \in U_{2n}(R,\Lambda)$, $k \neq \pm l$ and $i \neq \pm j$. Then

(i) *$T_{kl}(\sigma_{ij})$ is a product of* 160 *elementary unitary σ-conjugates,*

(ii) *$T_{kl}(\sigma_{i,-i})$ is a product of* 320 *elementary unitary σ-conjugates,*

(iii) *$T_{kl}(\sigma_{ii} - \sigma_{jj})$ is a product of* 480 *elementary unitary σ-conjugates,*

(iv) *$T_{kl}(\sigma_{ii} - \sigma_{-i,-i})$ is a product of* 960 *elementary unitary σ-conjugates and*

(v) *$T_{k,-k}(\lambda^{-(\epsilon(k)+1)/2}|\sigma_{*j}|)$ is a product of* $1600n + 4004$ *elementary unitary σ-conjugates.*

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In the proof of the theorem above matrices like

$$\tau = T_{21}(\bar{\sigma}_{23}\sigma_{23})T_{31}(-\bar{\sigma}_{23}\sigma_{22})T_{3,-2}(\bar{\sigma}_{23}\sigma_{2,-1})T_{3,-3}(-\bar{\sigma}_{22}\sigma_{2,-1}+\bar{\lambda}\bar{\sigma}_{2,-1}\sigma_{22})$$

in the stabilizer of the second row of σ are used.

Corollary

Let H be a subgroup of $U_{2n}(R,\Lambda)$. Then H is normalized by $EU_{2n}(R,\Lambda)$ iff

$$EU_{2n}((R,\Lambda),(I,\Gamma)) \subseteq H \subseteq CU_{2n}((R,\Lambda),(I,\Gamma))$$

for some form ideal (I,Γ) of (R,Λ).

R. Preusser, *Sandwich classification for* $GL_n(R)$, $O_{2n}(R)$ *and* $U_{2n}(R,\Lambda)$ *revisited*, accepted by J. Group Theory

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Thank you!