A Duality for Bounded Distributive Lattices with Order-Inverting Operation

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Outline

Motivation

Inspired by philosophical linguistics – David Ripley, *Negation in Natural Language*, PhD Thesis, University of North Carolina, 2009.

Setting

- Classical conjunction and disjunction $\vee, \wedge$.
- Existence of 1 and 0.
- A non-classical negation $\neg$, defined by

$$\frac{\alpha \vdash \beta}{\neg\beta \vdash \neg\alpha}$$

Bounded distributive lattices

Definition (Lattice)

An ordered set **L** is called a *lattice* if $L \neq \varnothing$ and $\bigvee\{a, b\}$ and $\bigwedge\{a, b\}$ exist for each $a, b \in L$. A lattice **L** is said to be *bounded* if $1 := \bigvee L$ and $0 := \bigwedge L$ exist in L.

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Definition (Distributive lattice)

A *distributive lattice* is a lattice in which join and meet distribute over one another; that is, the distributivity laws hold:

$$(\forall a, b, c \in L) \quad a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c);$$
$$(\forall a, b, c \in L) \quad a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c).$$

Bounded distributive lattices

Definition (Lattice homomorphism)

Let **L** and **K** be lattices. A map $\varphi\colon L \to K$ is called a *lattice homomorphism* if for all $a, b \in L$ we have

$$\varphi(a \wedge b) = \varphi(a) \wedge \varphi(b), \qquad \varphi(a \vee b) = \varphi(a) \vee \varphi(b).$$

It is a *bounded lattice homomorphism* if it is a lattice homomorphism and it preserves the bounds; i.e.

$$\varphi(0) = 0, \qquad \varphi(1) = 1.$$

Bounded distributive lattices

Definition

We define $\mathcal{D}$ to be the category whose objects are the bounded distributive lattices and whose morphisms are the bounded lattice homomorphisms.

BDLNs

Definition

A *bounded distributive lattice with negation* (*BDLN*) is an algebra $\mathbf{L} = \langle L; \vee, \wedge, \neg, 0, 1 \rangle$ such that

- $\langle L; \vee, \wedge, 0, 1 \rangle$ is a bounded distributive lattice, and
- $\neg$ (*negation*) is an order-inverting unary operation; that is,

$$x \leqslant y \Longrightarrow \neg y \leqslant \neg x$$

BDLNs

Definition

We define $\mathcal{K}$ to be the category whose objects are the BDLNs and whose morphisms are the bounded lattice homomorphisms that preserve negation.

Hom-functors

We define

$$H_A = \mathcal{C}(-, A) \colon \mathcal{C}^{\mathrm{op}} \to \mathbf{Set}$$

On objects B in $\mathcal{C}$ by

$$H_A(B) = \mathcal{C}(B, A)$$

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On objects B in $\mathcal{C}$ by

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On morphisms $g\colon B' \to B$ in $\mathcal{C}$ by

$$H_A(g)\colon \mathcal{C}(B, A) \to \mathcal{C}(B', A)$$
$$x \mapsto x \circ g$$

Dual equivalence of categories

Definition (Dual equivalence of categories)

A *duality* or *dual equivalence of categories* $\mathcal{A}$ and $\mathcal{B}$ consists of a pair of contravariant functors $S\colon \mathcal{B} \to \mathcal{A}$ and $T\colon \mathcal{A} \to \mathcal{B}$ with natural isomorphisms $\eta\colon \mathrm{id}_{\mathcal{B}} \to TS$ and $\varepsilon\colon \mathrm{id}_{\mathcal{A}} \to ST$.

Priestley Spaces

Definition (Priestley space)

A *Priestley space* $\langle X; \leqslant, \mathscr{T}\rangle$ is a topological space equipped with a partial order such that

- the underlying topological space $\langle X; \mathscr{T}\rangle$ is compact;
- $\langle X; \leqslant, \mathscr{T}\rangle$ is totally order-disconnected; i.e. if $x \nleqslant y$, there exists a clopen up-set U such that $x \in U$ and $y \notin U$.

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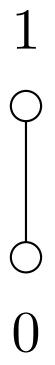
Definition

We define $\mathcal{P}$ to be the category whose objects are the Priestley spaces and whose morphisms are the continuous order-preserving maps.

Dualising objects

- $\underline{\mathbf{2}} := \langle \{0,1\}; \vee, \wedge, 0, 1 \rangle$ is the two-element chain in $\mathcal{D}$.
- $\underset{\sim}{\mathbf{2}} := \langle \{0,1\}, \leqslant, \mathscr{T} \rangle$ is the discretely topologised two-element chain – a Priestley space.



Priestley duality

- If $\mathbf{L}$ is a bounded distributive lattice, then $\mathcal{D}(\mathbf{L}, \underline{\mathbf{2}})$ is a closed subspace of $\bar{\mathbf{2}}^L$ and therefore forms a Priestley space

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- We define

$$K(\mathbf{X}) := \langle \mathcal{P}(\mathbf{X}, \bar{\mathbf{2}}); \vee, \wedge, 0, 1 \rangle \leqslant \underline{\mathbf{2}}^X$$

Priestley duality

- For a morphism $u\colon \mathbf{L} \to \mathbf{K}$ in $\mathcal{P}$, we define $H(u)\colon H(\mathbf{K}) \to H(\mathbf{L})$ via the hom-functor $\mathcal{D}(-, \underline{\mathbf{2}})$:

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- $\varepsilon_{\mathbf{X}}\colon \mathbf{X} \to HK(\mathbf{X})$ is defined similarly for Priestley spaces $\mathbf{X}$

Priestley duality

Theorem (Priestley Duality)

The mappings $H : \mathcal{D} \to \mathcal{P}$ and $K : \mathcal{P} \to \mathcal{D}$ are well-defined contravariant functors. Moreover,

$$\mathcal{D} \underset{KH}{\overset{\mathrm{id}_{\mathcal{D}}}{\rightrightarrows}} \mathcal{D} \;\; (\Downarrow \eta) \qquad \text{and} \qquad \mathcal{P} \underset{HK}{\overset{\mathrm{id}_{\mathcal{P}}}{\rightrightarrows}} \mathcal{P} \;\; (\Downarrow \varepsilon)$$

are natural isomorphisms, and hence the categories $\mathcal{D}$ and $\mathcal{P}$ are dually equivalent.

Functors

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- Define $J(\mathbf{P}) := \langle H_{\mathbf{2}}(\mathbf{P}), \leqslant\rangle$, where $\leqslant$ is the pointwise order on $H_{\mathbf{2}}(\mathbf{P}) = \mathbf{Pos}(\mathbf{P}, \mathbf{2})$.

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- Define forgetful functor $G\colon \mathcal{P} \to \mathbf{Pos}$ by $\langle X; \leqslant, \mathscr{T}\rangle \mapsto \langle X; \leqslant\rangle$ and on morphisms by $\varphi \mapsto \varphi$.

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- Define forgetful functor $G\colon \mathcal{P} \to \mathbf{Pos}$ by $\langle X; \leqslant, \mathscr{T}\rangle \mapsto \langle X; \leqslant\rangle$ and on morphisms by $\varphi \mapsto \varphi$.
- Finally, define $Z := J \circ {}^{\partial} \circ T \circ K\colon \mathcal{P} \to \mathbf{Pos}$.

Categories $\mathcal{S}$ and $\mathcal{K}$

- Objects of $\mathcal{S}$ are quadruples $\mathbf{S} = \langle S; \leqslant, \mathscr{T}, \mathcal{N} \rangle$, where $\mathbf{S}^\flat := \langle S; \leqslant, \mathscr{T} \rangle$ is a Priestley space and $\mathcal{N}$ is an order-preserving map from $G(\mathbf{S}^\flat) \to Z(\mathbf{S}^\flat)$ with the property that for each $\alpha \in \mathcal{P}(\mathbf{S}^\flat, \bar{\mathbf{2}})$, the map

$$n_\alpha \colon S \to \{0, 1\}, \quad x \mapsto \mathcal{N}(x)(\alpha),$$

is a $\mathcal{P}$-morphism $n_\alpha \colon \mathbf{S}^\flat \to \bar{\mathbf{2}}$.

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  is a $\mathcal{P}$-morphism $n_\alpha \colon \mathbf{S}^\flat \to \underline{\mathbf{2}}$.
- $h \colon \mathbf{S} \to \mathbf{T}$ is an $\mathcal{S}$-morphism if it is a $\mathcal{P}$-morphism and the diagram below commutes.

$$\begin{array}{ccc} G(\mathbf{S}^\flat) & \xrightarrow{G(h^\flat)} & G(\mathbf{T}^\flat) \\ \Big\downarrow \mathcal{N}^{\mathbf{S}} & & \Big\downarrow \mathcal{N}^{\mathbf{T}} \\ Z(\mathbf{S}^\flat) & \xrightarrow[Z(h^\flat)]{} & Z(\mathbf{T}^\flat) \end{array}$$

Duality for $\mathcal{S}$ and $\mathcal{K}$

Define $D\colon \mathcal{K} \to \mathcal{S}$ as follows:

- On an object $\mathbf{L}$ in $\mathcal{K}$ by

$$D(\mathbf{L}) := \langle \mathcal{D}(\mathbf{L}^\flat, \underline{\mathbf{2}}); \leqslant, \mathscr{T}, \mathcal{N} \rangle$$

where $\langle \mathcal{D}(\mathbf{L}^\flat, \underline{\mathbf{2}}); \leqslant, \mathscr{T} \rangle = H(\mathbf{L}^\flat)$ and $\mathcal{N}$ is defined by

$$\mathcal{N}(x)(\alpha) := x(\neg(\eta_{\mathbf{L}^\flat}^{-1}(\alpha)))$$

for all $x \in \mathcal{D}(\mathbf{L}^\flat, \underline{\mathbf{2}})$ and all $\alpha \in \mathcal{P}(H(\mathbf{L}^\flat), \bar{\mathbf{2}})$.

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- On a morphism $u\colon \mathbf{L} \to \mathbf{K}$ in $\mathcal{K}$ by

$$D(u)\colon D(\mathbf{K}) \to D(\mathbf{L}), \qquad D(u)(x) := x \circ u$$

Duality for $\mathcal{S}$ and $\mathcal{K}$

Define $E\colon \mathcal{S} \to \mathcal{K}$ as follows:

- On an object $\mathbf{S}$ in $\mathcal{S}$ by

$$E(\mathbf{S}) := \langle \mathcal{P}(\mathbf{S}^\flat, \bar{\mathbf{2}}); \vee, \wedge, \neg, 0, 1 \rangle$$

where $\langle \mathcal{P}(\mathbf{S}^\flat, \underline{\mathbf{2}}); \vee, \wedge, 0, 1 \rangle = K(\mathbf{S}^\flat)$ and $\neg$ is defined by

$$(\neg\alpha)(x) = \mathcal{N}(x)(\alpha)$$

for all $x \in S$ and all $\alpha \in \mathcal{P}(\mathbf{S}^\flat, \bar{\mathbf{2}})$.

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- On a morphism $h\colon \mathbf{S} \to \mathbf{T}$ in $\mathcal{S}$ by

$$E(h)\colon E(\mathbf{T}) \to E(\mathbf{S}), \qquad E(h)(\alpha) := \alpha \circ h$$

Duality for $\mathcal{S}$ and $\mathcal{K}$

We define mappings $\eta_{\mathbf{L}} : \mathbf{L} \to ED(\mathbf{L})$ and $\varepsilon_{\mathbf{S}} : \mathbf{S} \to DE(\mathbf{S})$ in the same way as with Priestley duality. We then have

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Theorem

The mappings $D \colon \mathcal{K} \to \mathcal{S}$ and $E \colon \mathcal{S} \to \mathcal{K}$ are well-defined contravariant functors. Moreover,

$$\mathcal{K} \underset{ED}{\overset{\mathrm{id}_{\mathcal{K}}}{\rightrightarrows}} \mathcal{K} \;\; (\Downarrow \eta) \quad \text{and} \quad \mathcal{S} \underset{DE}{\overset{\mathrm{id}_{\mathcal{S}}}{\rightrightarrows}} \mathcal{S} \;\; (\Downarrow \varepsilon)$$

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Thank you for listening.