

A Duality for Bounded Distributive Lattices with Order-Inverting Operation

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Outline

- 1 Background
 - Bounded Distributive Lattices
 - Bounded Distributive Lattices with Negation
 - Dual Equivalence of Categories
- 2 Priestley Duality
- 3 Duality for Bounded Distributive Lattices with Negation

Motivation

Inspired by philosophical linguistics – David Ripley, *Negation in Natural Language*, PhD Thesis, University of North Carolina, 2009.

Setting

- Classical conjunction and disjunction \vee, \wedge .
- Existence of 1 and 0.
- A non-classical negation \neg , defined by

$$\frac{\alpha \vdash \beta}{\neg\beta \vdash \neg\alpha}$$

Bounded distributive lattices

Definition (Lattice)

An ordered set \mathbf{L} is called a *lattice* if $L \neq \emptyset$ and $\bigvee\{a, b\}$ and $\bigwedge\{a, b\}$ exist for each $a, b \in L$. A lattice \mathbf{L} is said to be *bounded* if $1 := \bigvee L$ and $0 := \bigwedge L$ exist in L .

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Definition (Distributive lattice)

A *distributive lattice* is a lattice in which join and meet distribute over one another; that is, the distributivity laws hold:

$$(\forall a, b, c \in L) \quad a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c);$$

$$(\forall a, b, c \in L) \quad a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c).$$

Bounded distributive lattices

Definition (Lattice homomorphism)

Let \mathbf{L} and \mathbf{K} be lattices. A map $\varphi: L \rightarrow K$ is called a *lattice homomorphism* if for all $a, b \in L$ we have

$$\varphi(a \wedge b) = \varphi(a) \wedge \varphi(b), \quad \varphi(a \vee b) = \varphi(a) \vee \varphi(b).$$

It is a *bounded lattice homomorphism* if it is a lattice homomorphism and it preserves the bounds; i.e.

$$\varphi(0) = 0, \quad \varphi(1) = 1.$$

Bounded distributive lattices

Definition

We define \mathcal{D} to be the category whose objects are the bounded distributive lattices and whose morphisms are the bounded lattice homomorphisms.

BDLNs

Definition

A *bounded distributive lattice with negation (BDLN)* is an algebra $\mathbf{L} = \langle L; \vee, \wedge, \neg, 0, 1 \rangle$ such that

- ▶ $\langle L; \vee, \wedge, 0, 1 \rangle$ is a bounded distributive lattice, and
- ▶ \neg (*negation*) is an order-inverting unary operation; that is,

$$x \leq y \implies \neg y \leq \neg x$$

BDLNs

Definition

We define \mathcal{K} to be the category whose objects are the BDLNs and whose morphisms are the bounded lattice homomorphisms that preserve negation.

Hom-functors

We define

$$H_A = \mathcal{C}(-, A): \mathcal{C}^{\text{op}} \rightarrow \mathbf{Set}$$

On objects B in \mathcal{C} by

$$H_A(B) = \mathcal{C}(B, A)$$

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On objects B in \mathcal{C} by

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On morphisms $g: B' \rightarrow B$ in \mathcal{C} by

$$\begin{aligned} H_A(g): \mathcal{C}(B, A) &\rightarrow \mathcal{C}(B', A) \\ x &\mapsto x \circ g \end{aligned}$$

Dual equivalence of categories

Definition (Dual equivalence of categories)

A *duality* or *dual equivalence of categories* \mathcal{A} and \mathcal{B} consists of a pair of contravariant functors $S: \mathcal{B} \rightarrow \mathcal{A}$ and $T: \mathcal{A} \rightarrow \mathcal{B}$ with natural isomorphisms $\eta: \text{id}_{\mathcal{B}} \rightarrow TS$ and $\varepsilon: \text{id}_{\mathcal{A}} \rightarrow ST$.

Priestley Spaces

Definition (Priestley space)

A *Priestley space* $\langle X; \leq, \mathcal{T} \rangle$ is a topological space equipped with a partial order such that

- ▶ the underlying topological space $\langle X; \mathcal{T} \rangle$ is compact;
- ▶ $\langle X; \leq, \mathcal{T} \rangle$ is totally order-disconnected; i.e. if $x \not\leq y$, there exists a clopen up-set U such that $x \in U$ and $y \notin U$.

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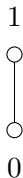
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Definition

We define \mathcal{P} to be the category whose objects are the Priestley spaces and whose morphisms are the continuous order-preserving maps.

Dualising objects

- ▶ $\underline{\mathbf{2}} := \langle \{0, 1\}; \vee, \wedge, 0, 1 \rangle$ is the two-element chain in \mathcal{D} .
- ▶ $\bar{\mathbf{2}} := \langle \{0, 1\}, \leq, \mathcal{T} \rangle$ is the discretely topologised two-element chain – a Priestley space.



Priestley duality

- ▶ If \mathbf{L} is a bounded distributive lattice, then $\mathcal{D}(\mathbf{L}, \mathbf{2})$ is a closed subspace of $\bar{\mathbf{2}}^L$ and therefore forms a Priestley space

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$$H(\mathbf{L}) := \langle \mathcal{D}(\mathbf{L}, \mathbf{2}); \mathcal{T}, \leq \rangle \leq \bar{\mathbf{2}}^L$$

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- ▶ We define

$$K(\mathbf{X}) := \langle \mathcal{P}(\mathbf{X}, \bar{\mathbf{2}}); \vee, \wedge, 0, 1 \rangle \leq \underline{\mathbf{2}}^X$$

Priestley duality

- ▶ For a morphism $u: \mathbf{L} \rightarrow \mathbf{K}$ in \mathcal{P} , we define $H(u): H(\mathbf{K}) \rightarrow H(\mathbf{L})$ via the hom-functor $\mathcal{D}(-, \mathbf{2})$:

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- ▶ $\varepsilon_{\mathbf{X}}: \mathbf{X} \rightarrow HK(\mathbf{X})$ is defined similarly for Priestley spaces \mathbf{X}

Priestley duality

Theorem (Priestley Duality)

The mappings $H: \mathcal{D} \rightarrow \mathcal{P}$ and $K: \mathcal{P} \rightarrow \mathcal{D}$ are well-defined contravariant functors. Moreover,

$$\begin{array}{ccc} \mathcal{D} & \begin{array}{c} \xrightarrow{\text{id}_{\mathcal{D}}} \\ \Downarrow \eta \\ \xrightarrow{KH} \end{array} & \mathcal{D} \end{array} \quad \text{and} \quad \begin{array}{ccc} \mathcal{P} & \begin{array}{c} \xrightarrow{\text{id}_{\mathcal{P}}} \\ \Downarrow \varepsilon \\ \xrightarrow{HK} \end{array} & \mathcal{P} \end{array}$$

are natural isomorphisms, and hence the categories \mathcal{D} and \mathcal{P} are dually equivalent.

Functors

- ▶ A covariant functor $\partial: \mathbf{Pos} \rightarrow \mathbf{Pos}$ defined on objects by $\mathbf{P} \mapsto \mathbf{P}^\partial$ and on morphisms by $\varphi \mapsto \varphi$.

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- ▶ Define $J(\mathbf{P}) := \langle H_2(\mathbf{P}), \leq \rangle$, where \leq is the pointwise order on $H_2(\mathbf{P}) = \mathbf{Pos}(\mathbf{P}, \mathbf{2})$.

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- ▶ Defining $J(\varphi) := H_2(\varphi)$ for morphisms φ in \mathbf{Pos} gives a well-defined contravariant functor $J: \mathbf{Pos} \rightarrow \mathbf{Pos}$.

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- ▶ Define forgetful functor $G: \mathcal{P} \rightarrow \mathbf{Pos}$ by $\langle X; \leq, \mathcal{T} \rangle \mapsto \langle X; \leq \rangle$ and on morphisms by $\varphi \mapsto \varphi$.

Functors

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- ▶ Define $J(\mathbf{P}) := \langle H_2(\mathbf{P}), \leq \rangle$, where \leq is the pointwise order on $H_2(\mathbf{P}) = \mathbf{Pos}(\mathbf{P}, \mathbf{2})$.
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- ▶ Define forgetful functor $G: \mathcal{P} \rightarrow \mathbf{Pos}$ by $\langle X; \leq, \mathcal{T} \rangle \mapsto \langle X; \leq \rangle$ and on morphisms by $\varphi \mapsto \varphi$.
- ▶ Finally, define $Z := J \circ \partial \circ T \circ K: \mathcal{P} \rightarrow \mathbf{Pos}$.

Categories \mathcal{S} and \mathcal{K}

- Objects of \mathcal{S} are quadruples $\mathbf{S} = \langle S; \leq, \mathcal{T}, \mathcal{N} \rangle$, where $\mathbf{S}^b := \langle S; \leq, \mathcal{T} \rangle$ is a Priestley space and \mathcal{N} is an order-preserving map from $G(\mathbf{S}^b) \rightarrow Z(\mathbf{S}^b)$ with the property that for each $\alpha \in \mathcal{P}(\mathbf{S}^b, \bar{\mathbf{2}})$, the map

$$n_\alpha: S \rightarrow \{0, 1\}, \quad x \mapsto \mathcal{N}(x)(\alpha),$$

is a \mathcal{P} -morphism $n_\alpha: \mathbf{S}^b \rightarrow \bar{\mathbf{2}}$.

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- ▶ $h: \mathbf{S} \rightarrow \mathbf{T}$ is an \mathcal{S} -morphism if it is a \mathcal{P} -morphism and the diagram below commutes.

$$\begin{array}{ccc} G(\mathbf{S}^b) & \xrightarrow{G(h^b)} & G(\mathbf{T}^b) \\ \mathcal{N}^{\mathbf{S}} \downarrow & & \downarrow \mathcal{N}^{\mathbf{T}} \\ Z(\mathbf{S}^b) & \xrightarrow{Z(h^b)} & Z(\mathbf{T}^b) \end{array}$$

Duality for \mathcal{S} and \mathcal{K}

Define $D: \mathcal{K} \rightarrow \mathcal{S}$ as follows:

- ▶ On an object \mathbf{L} in \mathcal{K} by

$$D(\mathbf{L}) := \langle \mathcal{D}(\mathbf{L}^b, \underline{\mathbf{2}}); \leq, \mathcal{F}, \mathcal{N} \rangle$$

where $\langle \mathcal{D}(\mathbf{L}^b, \underline{\mathbf{2}}); \leq, \mathcal{F} \rangle = H(\mathbf{L}^b)$ and \mathcal{N} is defined by

$$\mathcal{N}(x)(\alpha) := x(\neg(\eta_{\mathbf{L}^b}^{-1}(\alpha)))$$

for all $x \in \mathcal{D}(\mathbf{L}^b, \underline{\mathbf{2}})$ and all $\alpha \in \mathcal{P}(H(\mathbf{L}^b), \bar{\mathbf{2}})$.

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- ▶ On a morphism $u: \mathbf{L} \rightarrow \mathbf{K}$ in \mathcal{K} by

$$D(u): D(\mathbf{K}) \rightarrow D(\mathbf{L}), \quad D(u)(x) := x \circ u$$

Duality for \mathcal{S} and \mathcal{K}

Define $E: \mathcal{S} \rightarrow \mathcal{K}$ as follows:

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$$E(\mathbf{S}) := \langle \mathcal{P}(\mathbf{S}^b, \bar{\mathbf{2}}); \vee, \wedge, \neg, 0, 1 \rangle$$

where $\langle \mathcal{P}(\mathbf{S}^b, \underline{\mathbf{2}}); \vee, \wedge, 0, 1 \rangle = K(\mathbf{S}^b)$ and \neg is defined by

$$(\neg\alpha)(x) = \mathcal{N}(x)(\alpha)$$

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- ▶ On a morphism $h: \mathbf{S} \rightarrow \mathbf{T}$ in \mathcal{S} by

$$E(h): E(\mathbf{T}) \rightarrow E(\mathbf{S}), \quad E(h)(\alpha) := \alpha \circ h$$

Duality for \mathcal{S} and \mathcal{K}

We define mappings $\eta_{\mathbf{L}}: \mathbf{L} \rightarrow ED(\mathbf{L})$ and $\varepsilon_{\mathbf{S}}: \mathbf{S} \rightarrow DE(\mathbf{S})$ in the same way as with Priestley duality. We then have

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Theorem

The mappings $D: \mathcal{K} \rightarrow \mathcal{S}$ and $E: \mathcal{S} \rightarrow \mathcal{K}$ are well-defined contravariant functors. Moreover,

$$\mathcal{K} \begin{array}{c} \xrightarrow{\text{id}_{\mathcal{K}}} \\ \Downarrow \eta \\ \xrightarrow{ED} \end{array} \mathcal{K} \quad \text{and} \quad \mathcal{S} \begin{array}{c} \xrightarrow{\text{id}_{\mathcal{S}}} \\ \Downarrow \varepsilon \\ \xrightarrow{DE} \end{array} \mathcal{S}$$

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Thank you for listening.